Performance Based Learning and Assessment Task

Skate Ramp

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
The students will design a skateboard ramp which they will graph as a piecewise function, given certain parameters regarding function values, limits, and continuity. Students will have some pre-tasks to complete which will prepare them for the activity.

II. UNIT AUTHOR:
Cynthia Gillespie, Staunton River High School, Bedford County Schools
Ashley Swandby, James River High School, Botetourt County Schools
Linda Woodford, Franklin County High School, Franklin County Schools

III. COURSE:
Math Analysis

IV. CONTENT STRAND:
Algebra: Functions

V. OBJECTIVES:
The student will be able to:
- Identify function values from a graph
- Find the limit of a function as it approaches either a finite number or infinity, from a graph
- Find the zeros of a function from a graph
- Describe the continuity of a function at a given x-value, from a graph
- Use interval notation to describe where a function is increasing or decreasing, from a graph
- Draw the graph of a function, given parameters involving function values, limits, and continuity
- Write the equation of a piecewise defined function from its graph

VI. REFERENCE/RESOURCE MATERIALS:
Pre-Task worksheet and Skate Ramp Design Guidelines, paper to perform calculations, graph paper, poster board, and straightedge. Optional resources: graphing calculator, computer programs such as Desmos or Geogebra.

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will complete a self-assessment checklist based on a provided rubric. The teacher will use the same rubric to assess student performance based on correct mathematical computations, correct graphical representations, adherence to prescribed parameters, and a neat presentation of results.

VIII. EVALUATION CRITERIA:
Assessment lists, corresponding rubric, and a sample benchmark are included.

IX. INSTRUCTIONAL TIME:
This activity will take two class periods.

Skate Ramp

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Algebra: Functions

Mathematical Objective(s)
The overall mathematical goal of this activity is for students to design a skateboard ramp and graph it as a piecewise defined function. They will complete a pre-task activity in which they will review using a graph to find: function values, zeros, limits, intervals where the function is increasing or decreasing, and points of discontinuity. An additional pre-task activity will have students create a piecewise defined function given parameters involving specific function values and zeros, limits and points of discontinuity. Then students will design their own skateboard ramp to be a piecewise function within certain parameters and find the equation of their piecewise function. Their product will be an equation and graph of a piecewise function.

Related SOL

- MA.1 (The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercept, asymptotes, and intervals for which the function is increasing or decreasing. Graphing utilities will be used to investigate and verify these characteristics.)
- MA.3 (The student will investigate and describe the continuity of functions, using graphs and algebraic methods.)
- MA.7 (The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.)
- AII.6 (The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.)
- AFDA.1 (The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:
  a) continuity;
  c) domain and range;
  d) zeros;
  f) intervals in which the function is increasing/decreasing;
  g) end behaviors;and
  h) asymptotes.)
- AFDA.2 (The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).)
- AFDA.4 (The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.)

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationship
- Draw reasonable conclusions about a situation being modeled
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Interpret representations of functions of two variables
- Use symbolic algebra to represent and explain mathematical relationships

Materials/Resources

Students will need the following materials to complete the activity:

- Pre-Task worksheet and Skate Ramp Design Guidelines
- Paper to perform calculations
- Graph paper
- Poster board
- Straightedge
- Computer Programs such as Geogebra or Desmos (optional)
- Graphing calculator (optional)

Assumption of Prior Knowledge

Students should have basic knowledge of how to graph linear, absolute value, step, polynomial, rational and exponential functions, with labeled axes, using an appropriate scale. They should know how to analyze a piecewise defined graph to include: finding function values, zeros, limits, intervals where the function is increasing or decreasing, and points of discontinuity. They should also be able to write a piecewise function from the provided graph, using appropriate function notation.

As students design their skateboard ramps, they should discuss what causes points of discontinuity in graphs. They will talk about limits and how to make a function take on a particular value, and what type of function is best suited to each piece. They should also be considering the feasibility of their final design as an actual skateboard ramp, and should be discussing whether a person could actually traverse it as they have designed it.

Students may find it difficult to fit their graph to given criteria or to write equations for each piece of their function from the graph. The purpose of the pre-task activities is to expose students to the type of thinking required to complete the task and to give them the opportunity to practice and ask questions before they design their own skateboard ramp. Any groups needing help can be given suggestions.

Introduction: Setting Up the Mathematical Task

- The teacher will introduce the task by asking students "How many of you like skate boarding? Have you ever thought about how the different ramps are made?" In this activity, students will apply their knowledge of piecewise functions in order to design a skateboard ramp. Students should be comfortable with various function families, graphing functions, writing equations of functions, and limits.

- This lesson should take two class periods. During the first day, the teacher will prompt students to recall their knowledge of limits, continuity, and functions by completing Pre-Task 1 and Pre-Task 2. In Pre-Task 1, the teacher will provide students with a graph of a piecewise function containing an absolute value function, quadratic function, and a rational function. The students will determine limits, zeros, points of discontinuity, and intervals of increasing/decreasing. In Pre-Task 2, the student will create a piecewise function using several different types of functions that have particular limits, values, and continuity requirements. The students should apply their knowledge of graphing functions and writing equations of functions to complete this task. In Pre-Task 2, students may struggle to meet the requirements of the function. Some prompts could include having them start with one section of the function, and then apply transformation rules to have the next function be continuous or discontinuous that the given point. Both tasks could be done individually with feedback, either from the teacher or by having students present their solutions to each other and receive comments and feedback. The tasks could also be completed in pairs and then presented to the class, another group, or the teacher for feedback. The teacher or students should check the functions to ensure that they do in fact meet the criteria so that students are prepared for the task. In order to assist the teacher with checking the functions, having the students work in pairs would be preferred in order for students to check each other's work.

- For the task of creating the skateboard ramp, the students should be comfortable with piecewise functions and be able to create a function to meet requirements for values and continuity as in the pre-tasks. Teachers could have students search for pictures of skate parks to see the types of ramps that are used. Students could also watch videos of skaters using various ramps to do stunts. The teacher will then ask the students to create a ramp for the city planners to meet certain criteria for fun and safety. This task should be completed in pairs or in groups of 3 to 4 to encourage students to discuss ideas and make decisions as a group. Students should present their poster to the class by providing the graphs, equations used, and rationale for their choice of ramp.

- Questions to prompt students: What would skaters want in a ramp? How can you use various functions to achieve a fun and safe ramp? How can you use the pre-tasks to help you design the ramp?

Student Exploration

- Students complete Pre-Tasks 1 and 2 individually or in pairs, then receive feedback.
- Students complete the design task in pairs.
- Students present their graphs, equations, and rationale for their function representing a fun and safe ramp for the skate park.

Student/Teacher Actions:

- In the pre-tasks, students should be identifying features of a piecewise function and creating functions that have particular features. In the task, students should be creating a function that models a fun and safe skateboard ramp that meets particular criteria.
- During the pre-tasks and tasks, the teacher should ensure that the students are correctly identifying the features of the graphs, creating functions that appropriately meet the criteria, and creating a skateboard ramp that has the required features. The teacher can ask the students to justify their functions if the teacher identifies issues with the functions in order to help the students correct their own mistakes. Students may misidentify features or create functions that do not have the appropriate limit or function values or do not meet the continuity requirements.
- To bridge between the pre-tasks and the ramp task, the teacher could have students consider the graphs in the pre-tasks to determine if the functions would make a good ramp design. Have the students identify features that are good and features that are not good for a skateboard ramp. Consider things like height, elevation change, and continuity.
- This task can be done with or without the use of technology (on graph paper, or with calculators or computer graphing software). Teachers may wish to have students start without technology and use technology to check their solutions.

Monitoring Student Responses

Students should present both a graph and an equation for their functions in Pre-Task 2. For the skateboard ramp task, students should also present a graph and an equation that meets the objectives both mathematically and physically for a realistic skateboard ramp. Students should discuss their functions with each other to facilitate a conversation about what would make a function "work" for the requirements and the objectives. Students who are having difficulties should be prompted to start with one function over part of the domain and try to make a second function that would be continuous or discontinuous by checking function values at that point and making adjustments. Students could use technology to facilitate the adjustments. If students need an additional task, the students could find pictures of skate ramps on the internet and try to fit equations to them. Students will be provided with a rubric to assist them in meeting the task objectives.

Students will summarize their activity by presenting the graph and the equation of their graph to the class or the teacher. By using technology to graph, students will verify that their function meets the requirements of the function mathematically. The class could help the groups determine if the ramps appear to be good for skating.

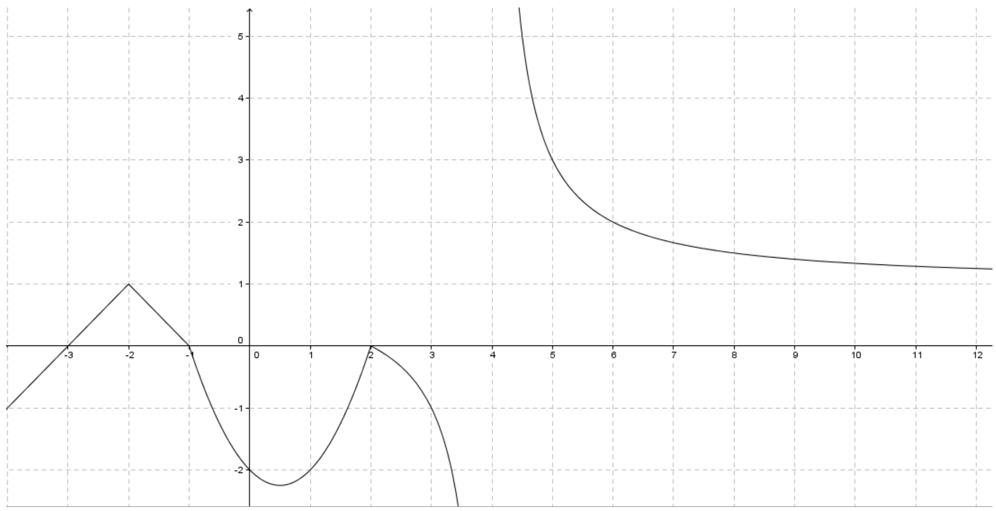
Assessment List and Benchmarks

Students will complete a skate ramp design based on the given guidelines. Students will self-assess their work using the rubric provided. The teacher will use the same rubric to assess the students' performance. Students will also present their ramp design to the class and discuss if they like the design or how it could possibly be changed. Students could provide feedback to each group by identifying one good aspect of the design and one suggestion to make it better.

Skate Ramp: Pre-Task 1 and 2

Pre-Task #1

Using the function graphed below, determine the following:



a) $f(0) =$

b) $\lim_{x \to -\infty} f(x) =$

c) the zeros of the function

d) the point(s) of discontinuity of the function

e) intervals where the function is increasing

Pre-Task #2:

Using at least three different types of functions (step, absolute value, polynomial, rational, or exponential), create a single piecewise defined function that has all of the following characteristics. You should write the equation in explicit form and provide a graph.

a) $\lim_{x \to -3} f(x) = \infty$

b) $\lim_{x \to \infty} f(x) = -\infty$

c) $\lim_{x \to -\infty} f(x) = 2$

d) $f(0) = 4$

e) $f(2) = 0$

f) has at least one jump discontinuity

Skate Ramp: Design Guidelines

The city planning board has decided to build a new skate park in your neighborhood. They have asked you to design the ramp so that it is fun and safe. In order to make it both fun and safe you should follow these guidelines and use your knowledge of functions to design the new ramp.

- 45 feet long
- at least two intervals of increasing
- at least two intervals of decreasing
- a maximum height of 3-10 feet
- at least three different types of functions (conics, polynomials, absolute value, exponential, rational)
- continuous everywhere
- transitions between functions reasonable for skating (consider the smoothness between functions)
- the height of the ramp should not change more than 5 vertical feet per horizontal foot (to avoid skaters in freefall)

The city planners need a cross-section graph of your design along with the function defined using piecewise notation.

Skate Ramp: Grading Rubric

#	Element Graded	0 points	1 point	2 points	Assessment	
					Self	Teacher
Pre-Task #1						
1	Completion	0 problems completed	4 or less problems completed	5-6 problems completed		
Pre-Task #2						
2	Display is Neat	Things are scribbled out in the equation and/or the graph	Lots of erasure marks in the equation and/or the graph	No erasure or scribble marks in the equation and/or the graph		
3	Use at least 3 different types of functions	Uses only 1 type of function	Uses at least 2 different types of functions	Uses a minimum of 3 different types of functions		
4	$f(0)=4$ and $f(2)=0$	Neither point is correct	Only 1 point is correct	Both points are correct		
5	$\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to-\infty} f(x) = 2$	Neither infinite limit is correct	Only 1 infinite limit is correct	Both infinite limits are correct		
6	$\lim_{x\to-3} f(x) = \infty$ and has at least one jump discontinuity	Neither discontinuity is correct	Only 1 discontinuity is correct	Both discontinuities are correct		
Skate Ramp Design						
7	Display is Neat	Things are scribbled out in the equation and/or the graph	Lots of erasure marks in the equation and/or the graph	No erasure or scribble marks in the equation and/or the graph		
8	Correct length and height	Is not the correct length or height	Either the correct length or the correct height	Is the correct length and the correct height		
9	Increasing/Decreasing	Has only 1 interval of increasing and 1 interval of decreasing	Has 1 interval of increasing and 2 intervals of decreasing (or vice versa)	Has at least 2 intervals of increasing and at least 2 intervals of decreasing		
10	Use at least 3 different types of functions	Uses only 1 type of function	Uses at least 2 different types of functions	Uses a minimum of 3 different types of functions		
11	Continuous	Not continuous at 2 or more locations	Not continuous at 1 location	Continuous everywhere		
12	Presentation	Does not present design to the class	Presents the design in an unclear and rushed fashion; not sufficient justification	Presents design to the class in a clear fashion and includes sufficient justification of design		
				TOTAL (out of 24)		

Skate Ramp: Benchmark

Pre-Task #1

a) $f(0) =$ -2

b) $\lim_{x \to -\infty} f(x) =$ -∞

c) The zeros of the function are at $x = -3, -1, 2$.

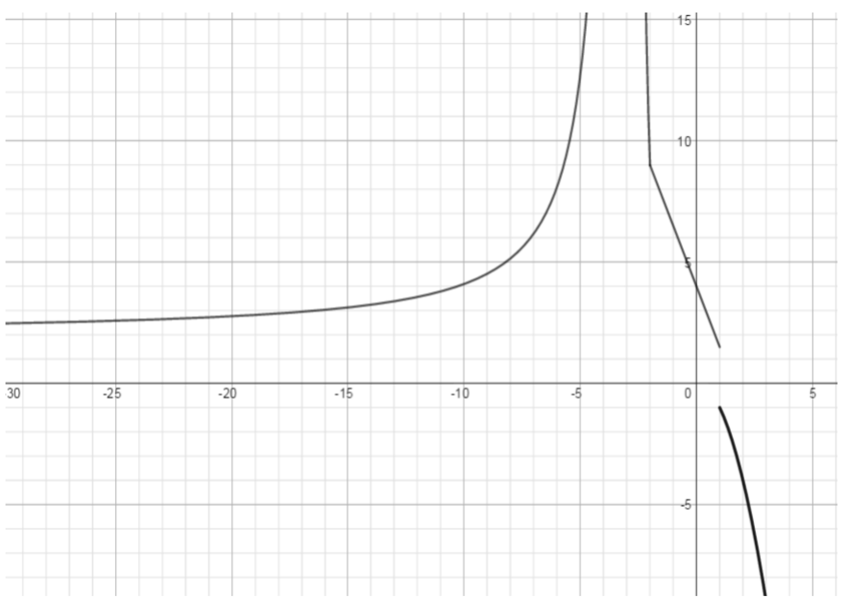
d) The point of discontinuity of this function is at $x = 4$.

e) This function is increasing on the following intervals $(-\infty, -2) \cup (-0.5, 2)$.

Pre-Task #2

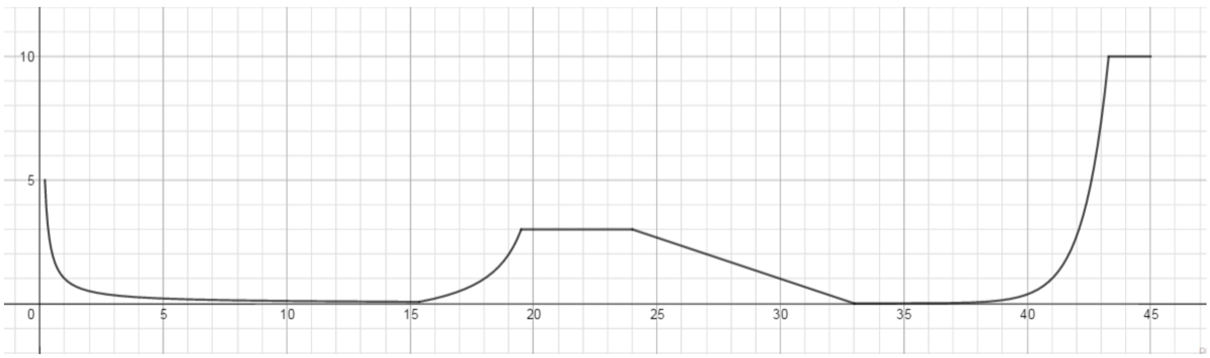
$$f(x) = \begin{cases} \dfrac{2x^2 + 1}{(x+3)^2} & x < -2 \\ -2.5x + 4 & -2 \le x \le 1 \\ -x^2 & x > 1 \end{cases}$$

Graphed using desmos.com



Skate Ramp Design:

$$f(x) = \begin{cases} \dfrac{1}{x} & \dfrac{1}{5} < x < 15.37 \\ -\dfrac{6}{x-21} - 1 & 15.37 < x \leq 19.5 \\ 3 & 19.5 < x \leq 24 \\ -\dfrac{1}{3}(x-15)+6 & 24 < x \leq 33 \\ e^{x-41} & 33 < x \leq 43.3 \\ 10 & 43.3 < x \leq 45 \end{cases}$$



On the left side of the ramp, the skater would go up and come back down, on the right side of the ramp, there would be a platform for the skater to stop.

http://www.skateparkguide.com/design_basics.html